Evidence for the Breakdown of the general Plane-Wave Field Emission Theory in Nanometric Emitters

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What is the state of the Field Emission (FE) theory at the moment

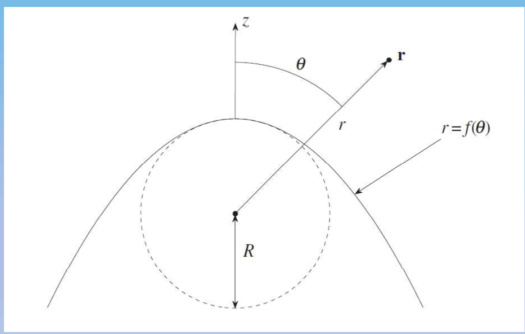
- The Murphy-Good (MG) theoretical updating (inclusion of the exchange and correlation interactions) of the Fowler-Nordheim (FN) theory has served successfully the FE community for 70 years.
- However, the MG equation holds theoretically for planar surfaces and practically for low curvature surfaces.
- Why??? It uses a linear electrostatic potential to calculate tunneling probabilities, valid only for planar-plate capacitor.
- However, emitters with as low radius as R=10-5nm are routinely made,
- How low curvature must the emitting surface have for the MG to be valid?
- Recent calculations of mine suggest R(apex)=30nm approximately.

What theory do we have below R<30nm???

- One can develop the next level of approximation i.e.
- 1) take into account the x^2 term in the electrostatic field and
- 2) use a spherical image potential instead of a planar one
- Work by myself and A. Kyritsakis

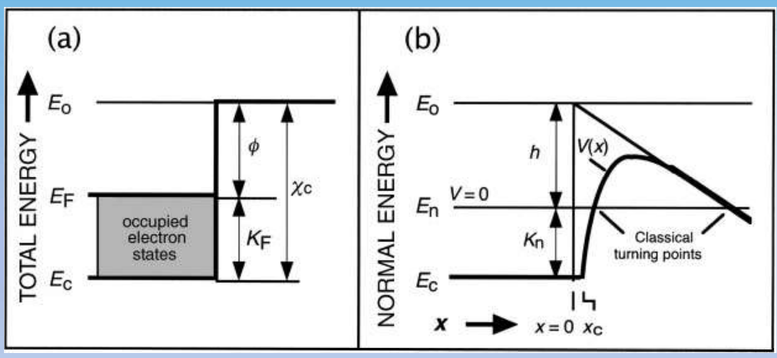
$$U(z) = \varphi - eFz - \frac{B}{z(1 + z/2R)} + \frac{eF}{R}z^2,$$

$$J = a\varphi^{-1}F^2\left(t(y) + \frac{\varphi}{eFR}\psi(y)\right)^{-2}\exp\left[-b\frac{\varphi^{3/2}}{F}\left(v(y) + \frac{\varphi}{eFR}\omega(y)\right)\right]$$



Previous Generalization of MG has a limit on how sharp the tip is

The MG equation is derived by using the Sommerfeld model i.e.by examining the electron states in a **rectangular quantum box** of depth Ef+φ. It then necessarily uses **plane waves** for the eigenstates. That cannot be true for electrons in ultra sharp tips.

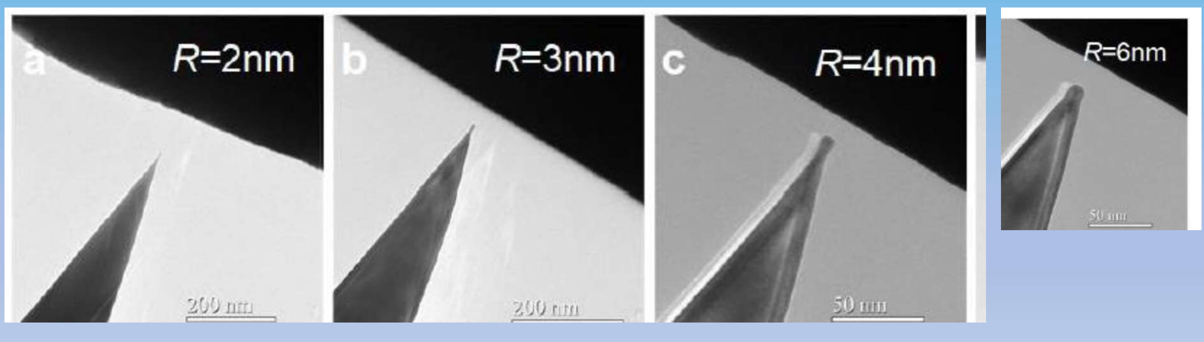


Justifying previous claim

- The wavelength of electrons at the Fermi level is λ_F =0.5nm approx/ly.
- How can you consider a surface of a radius of curvature R=0.65-3nm planar when the wavelength is 0.5nm-0.7nm.
- Ray theory of electromagnetism suggests that one needs surfaces with R **orders of magnitude larger than the wavelength.**
- **But let us see what most general calculations show.**
- Assume first that plane wave theory is correct and we have CFE, T=0.
- Furthermore, we will use the complete potential, no approximation, so that no doubt remains about approximations in the potential.

Fabricated emitters

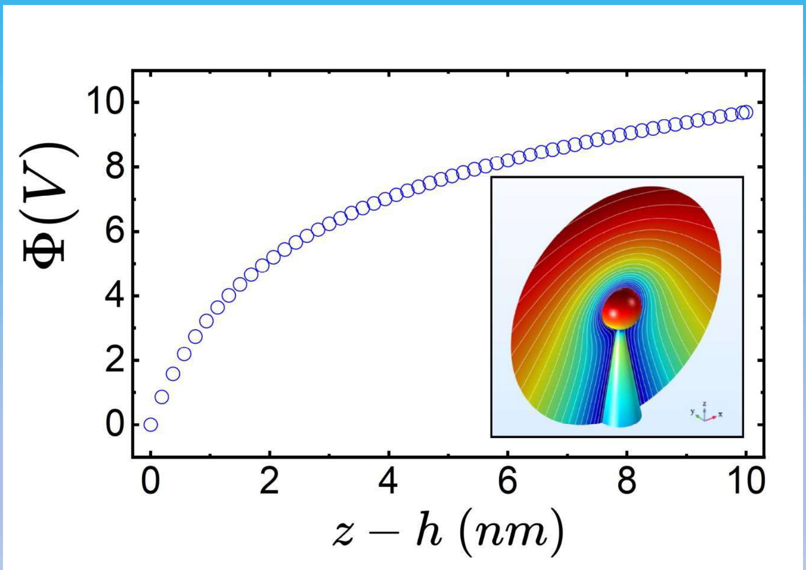
- Guodong Meng has manufactured tungsten emitters of varying R.
- Here are the nanometric ones. A spherical cap is left on the top of the emitter for all R. Only visible for R>=4nm. Remember that when I present the electrostatic calculations.



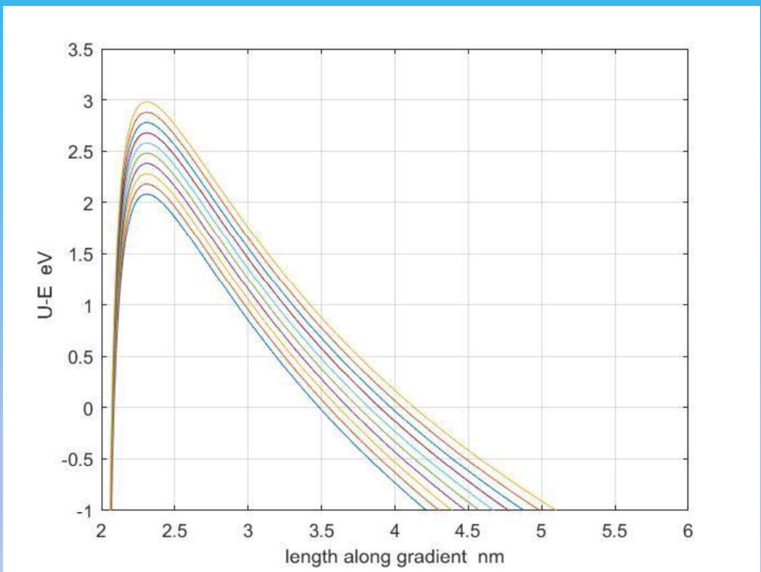
The most general theory assuming plane waves as wavefunctions of incident electrons is:

- $J = \frac{emk_BT}{2\pi^2\hbar^3}\int ln\left[1 + exp\left(\frac{E_F - E_n}{k_BT}\right)\right] D(E_n)\, dE_n$ (1)
- E_n are the normal to the barrier energies
- $D(E_n)$ are the transmission coefficients
- Remaining symbols have their conventional meaning
- $D(E_n) = 2\int_{z1}^{z2}\sqrt{\frac{2m_e}{h^2}(U(z) - E_n)}\, dz$,U(z)=tunneling potential
  $U(z)=E_F+\phi-eV_{electr}(z)-eV_{sphim}(z)$
- Current I=A*J, note A does not affect slope of FN plot.

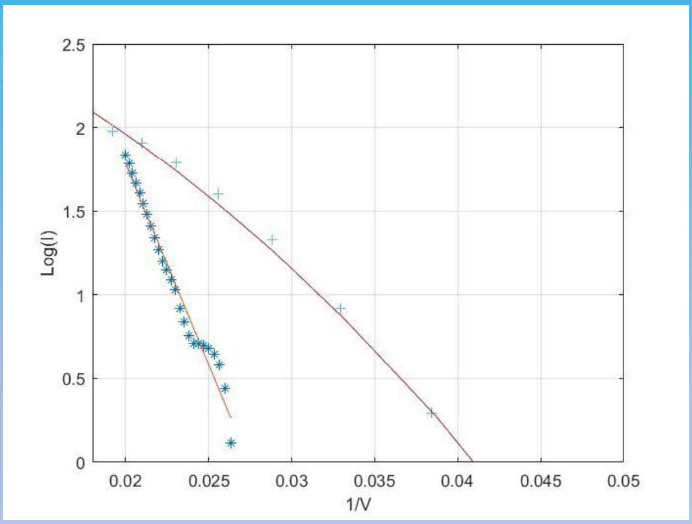
Electrostatic Potential for R=2nm,d=25nm



Tunneling Potentials at V_{anode}=13.02eV for several energies below the Fermi level



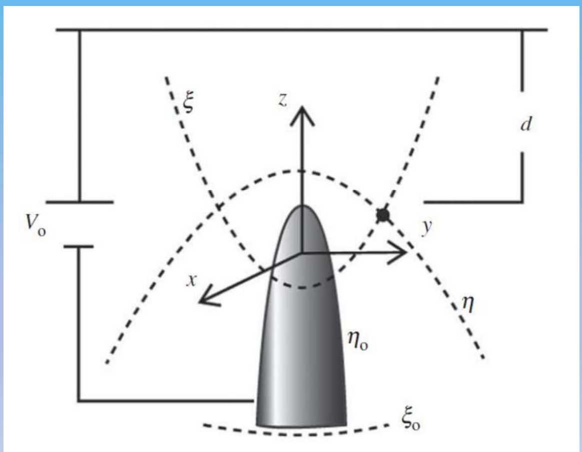
Compare experiment and planar theory
No Way that the two data sets agree



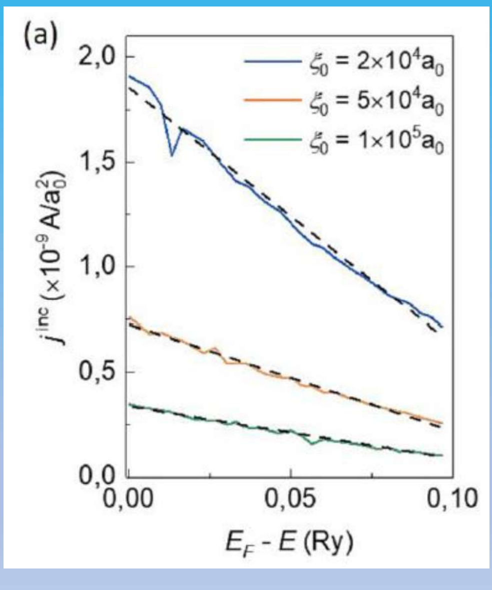
Elements of parabolic theory

- The emitter is treated as a paraboloid. The 3D Schroedinger equation is expressed in parabolic coordianates so that **the wavefronts of the wavefuctions agree with the emitter surface.**

1)Along φ we get m=integer.
2)Due to $\eta<\eta_0$ = nm, eigenstates are quantized, we get quantum number n =integer along η coor/te.
3)So the total energy of the states has the form $E = f(m, n, k_\xi)$, Hence an alternative labelling of the eigenstates is (E, m, n).



Constructing the supply function



1)The wavefunctions are labelled as Ψn,m,k_ξ(η, ξ, φ)= Ψn,m,E(η, ξ, φ).

2)Incident current densities j^{inc} can be calculated by the use of the relation

$$j^{inc}(n, m, E) = \left(\frac{eh}{m_e}\right)\Psi(n,m,E)\nabla\Psi(n,m,E)$$

3)All j^{inc} for m>0 can be ignored. $j^{inc}(E)$ almost constant with just a very weak (E_F -E) relation.

Transmission coefficient

- From the 3D Schroedinger equation **one can separate the 1D one in the direction normal to the barrier.** m=0 has been assumed. This reads as follows:

$$\frac{-\hbar^2}{2m_e}\frac{\partial^2\Psi}{\partial\eta^2}+\frac{1}{4}\left[V_{elst}(\eta)-\mathrm{E}-\frac{C(n,E)}{\eta}-\frac{\hbar^2}{2m_e\eta^2}\right]\Psi=0$$

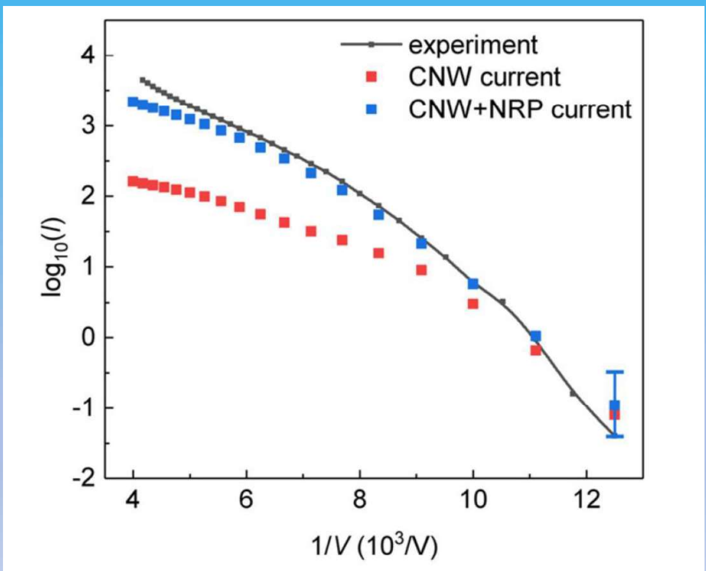
- Applying the JWKB method to this equation (the slowly varying condition has been checked) we get for the transmission coefficient normal to the barrier (4^{th} term in brackets negligible)
- $D_\eta(E)=\int_{z1}^{z2}\sqrt{\frac{2m_e}{h^2}(\mathrm{U(z)}-E)-\frac{C(n,E)}{\eta}})\,dz$, z axis is the ξ=0 contour,
- where $U(z)=E_F+\varphi$-elec/static term-image term

Transmitted current density

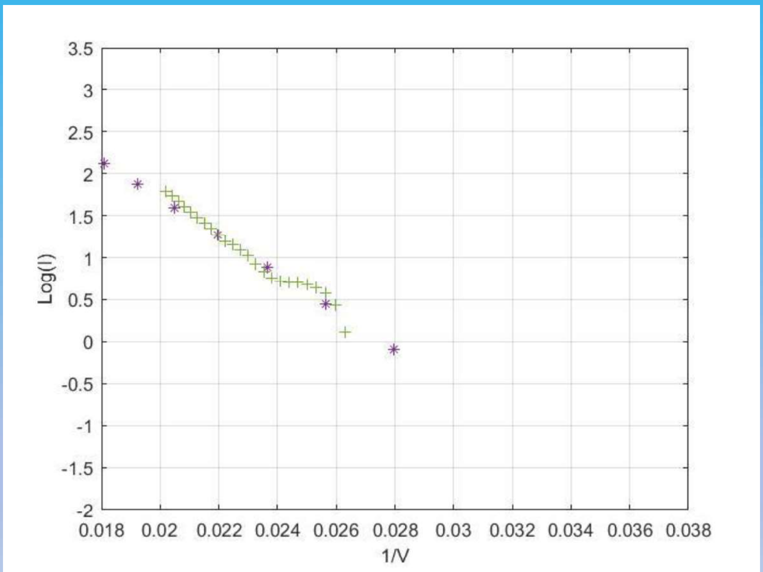
- $J^{trs}=\sum_{n,E} j_n^{inc}(E) D_\eta^n$ (E)

- The summation over E can be turned into an integration
- The quantum number n assumes only a limited number of values.
- Why??
- For C(n)>0 the $j_n^{inc}(E)$decreases with n increasing.
- For C(n)<0 transmission decreases with |C| increasing.
- C(n)/η acts like a forbidden region.
- The theory has been successfully applied to a carbon emitter of R=0.65nm, (Proc. R. Soc. 2025).

Ln(I) vs 1/V plots of a 0.65nm carbon emitter



Ln(I) vs 1/V of the 2nm emitter of G. Meng
Agreement of the slopes is excellent



Conclusions

- The most general field emission planar theory fails when R=a few nm.
- There are strong theoretical reasons for this failure: the emission area can not be considered planar because the electron wavelength is comparable to R.
- Field emission theory based on parabolic coordinates with wavefronts that match the emitting surface reproduces the experimental data of R=0.65nm and R=2nm experiments.
- More work is necessary.